

UNIT - 7

POLYPHASE CIRCUIT

☆ Polyphase System:- यह दो या दो से अधिक वोल्टेज^{का combination है} जो एक समान magnitude तथा frequency के हों किंतु एक दूसरे के साथ समान phase difference में होते हैं। अर्थात् यदि 3 voltage हैं तो आपस में 120° electrical angle से displaced रहेंगे।

$$\text{phase difference} = \frac{360^\circ \text{ electrical degrees}}{\text{No of phase}}$$

Advantage of 3 phase system over 1 ϕ system:-

- ① constant power:- 1 ϕ system में power जो deliver होता है वह pulsating होता है जबकि 3 ϕ system में power लगभग constant होता है।
- ② Higher Rating:- Rating (output) single phase machine की तुलना में ~~बड़ा~~ लगभग 1.5 गुना अधिक होता है जबकि दोनों मशीन का size बराबर हो तो पर भी।
- ③ Power Transmission Economics:- किसी fixed distance तक एक fixed power को प्रसारित करने में, 1 ϕ system में जो conducting material उपयोग होता है, 3 ϕ system में केवल उसका 75% conducting material की जरूरत पड़ती है।
- ④ Higher power factor & efficiency:- 3 ϕ system में उपयोग होने वाले मशीनों की efficiency तथा power factor अधिक होगी है तथा जब्त शक्ति भी होती है।

★ STAR CONNECTION :-

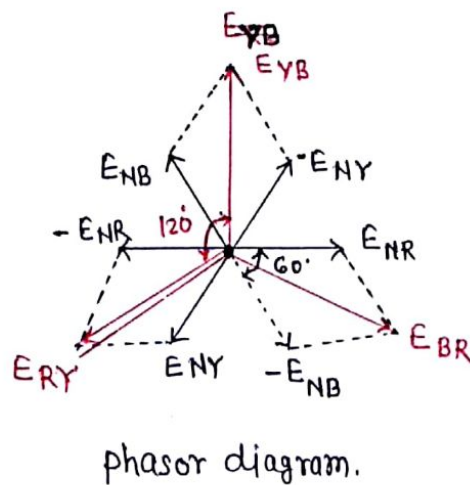
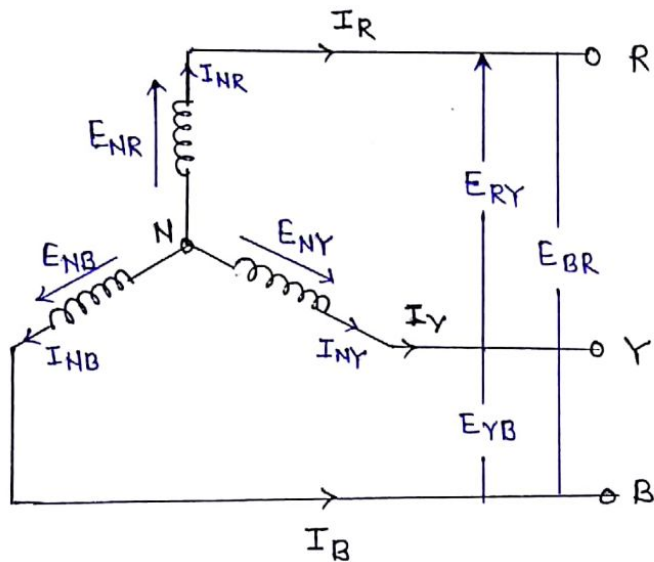
Que: In a three phase balanced star connected system, find the relation between :-

- (a) Line volt. & phase voltage
- (b) Line current & phase current
- (c) Total 3 ϕ power

किसी त्रिकोणीय समतुल्य स्टार कनेक्शन सिस्टम के लिए निम्न संबंध स्थापित कीजिए :-

- (a) लाइन वोल्टेज व फेज वोल्टेज में ।
- (b) लाइन करंट व फेज करंट में ।
- (c) कुल त्रिकोणीय शक्ति ।

Ans: -



① Relation b/w V_{ph} & V_L :



प्रत्येक phase में (winding) में वोल्टेज को phase voltage कहते हैं तथा Line conductor के across voltage को Line voltage (V_L) कहते हैं।

$$\text{Now } E_{NR} = E_{NY} = E_{NB} = E_{ph} \text{ (phase voltage)}$$

$$\text{LOOP NRYN से } \vec{E}_{NR} + \vec{E}_{RY} - \vec{E}_{NY} = 0$$

$$\vec{E}_{RY} = \vec{E}_{NY} - \vec{E}_{NR} \text{ (vector difference)}$$

$$E_{RY} = \sqrt{E_{NY}^2 + E_{NR}^2 + 2E_{NY}E_{NR}\cos 60^\circ}$$

$$E_L = \sqrt{E_{ph}^2 + E_{ph}^2 + 2E_{ph} \cdot E_{ph} \times 0.5}$$

$$= \sqrt{3E_{ph}^2}$$

$$E_L = \sqrt{3} E_{ph}$$

$$\text{where } E_{RY} = E_{YB} = E_{BR} = E_L \text{ (line voltage)}$$

अतः star connection में

$$\text{लाइन वोल्टेज} = \sqrt{3} \times \text{फेज वोल्टेज}$$

② Relation b/w I_L & I_{ph} :-

चित्र से स्पष्ट है कि phase winding तथा line conductor में same current flow होगा क्योंकि आपस में series में जुड़े हैं।

$$\text{अर्थात् } I_R = I_{NR}$$

$$I_Y = I_{NY}$$

$$I_B = I_{NB}$$

$$I_{NR} = I_{NY} = I_{NB} = \text{(phase current)} I_{ph}$$

$$\text{and } I_R = I_Y = I_B = \text{(line current)} I_L$$

अतः स्टार कनेक्शन में (I_L) Line current = phase current (I_{ph})

⑤ Power

Power = $3VI \cos \phi$ (जहाँ पर Volt. & current, phase Volt & phase current हैं)

$$\text{Line Power } (P_L) = 3 V_{ph} I_{ph} \cos \phi$$

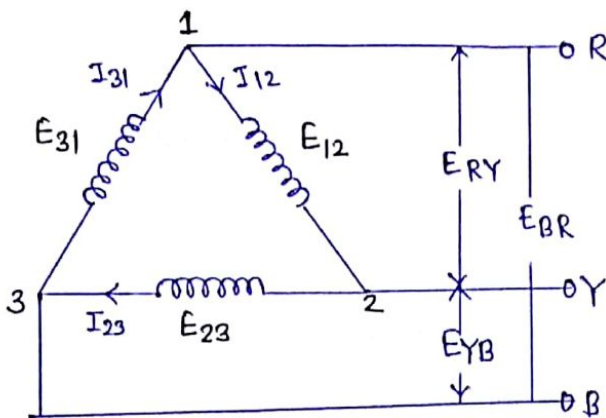
$$(I_{ph}) = I \text{ since } V_{ph} = \frac{V_L}{\sqrt{3}} \text{ तथा } I_{ph} = I_L$$

$$(P_L) = 3 \times \frac{V_L}{\sqrt{3}} \times I_L \cos \phi$$

$$P_L = \sqrt{3} V_L I_L \cos \phi$$

☆ DELTA CONNECTION :-

Q:- In 3ϕ balanced delta connected system, find relation between
(a) V_L and V_{ph} (b) I_L and I_{ph} (c) Total 3ϕ power



(a) Relation b/w V_{ph} and V_L

चित्र से स्पष्ट है कि Terminal 1 & 2 के across जो voltage होगा वह E_{RY} के बराबर होगा क्योंकि parallel में वोल्टेज same होगा।

$$\therefore E_{12} = E_{RY}$$

इसी प्रकार $E_{23} = E_{YB}$

तथा $E_{31} = E_{BR}$

जहाँ पर $E_{12} = E_{23} = E_{31} = E_{ph} = (\text{Phase Voltage})$

तथा $E_{RY} = E_{YB} = E_{BR} = E_L = (\text{Line Voltage})$

Hence in delta connection

$$V_{ph} = V_L \text{ OR } E_{ph} = E_L$$



② Relation b/w I_{ph} and I_L

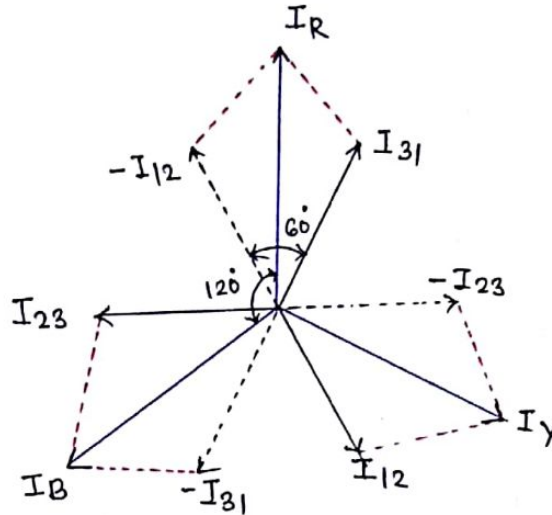


Fig: phasor diagram

$$I_{12} = I_{23} = I_{31} = I_{ph} = (\text{phase current})$$

Junction 1 में KCL लगाने पर

$$\bar{I}_{31} = \bar{I}_R + \bar{I}_{12}$$

$$\text{OR } \bar{I}_R = \bar{I}_{31} - \bar{I}_{12} \quad (\text{vector difference}) \quad \text{--- ①}$$

इसी तरह Junction 2 पर

$$\bar{I}_Y = \bar{I}_{12} - \bar{I}_{23}$$

तथा Junction 3 पर

$$I_B = \bar{I}_{23} - \bar{I}_{31}$$

$$I_R = \sqrt{I_{31}^2 + I_{12}^2 + 2 I_{31} I_{12} \cos 60^\circ}$$

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2 I_{ph} \cdot I_{ph} \cos 60^\circ}$$

($\because I_R = I_L =$
line current)

$$I_L = \sqrt{3 I_{ph}^2}$$

$$I_L = \sqrt{3} I_{ph}$$

Similarly $\bar{I}_Y = \bar{I}_{12} - \bar{I}_{23}$ or $I_L = \sqrt{3} I_{ph}$

$$\bar{I}_B = \bar{I}_{23} - \bar{I}_{31}$$
 or $I_L = \sqrt{3} I_{ph}$

अतः delta connection में

$$\text{Line current} = \sqrt{3} \times \text{phase current}$$

③ Power: -

$$1\phi \text{ system में Power} = VI \cos \phi$$

जहाँ पर $V =$ वोल्टेज of 1ϕ i.e. V_{ph}

$I =$ 1ϕ में current i.e. I_{ph}

अतः 3ϕ system के लिए

$$P = 3 V_{ph} I_{ph} \cos \phi \quad \text{--- (2)}$$

अतः Δ connection के लिए

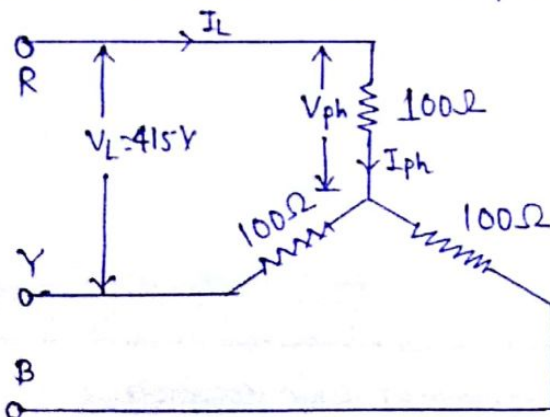
$$V_{ph} = V_L \text{ तथा } I_{ph} = \frac{I_L}{\sqrt{3}}$$

$$P = 3 \times V_L \times \frac{I_L}{\sqrt{3}} \cdot \cos \phi$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

Que: Three 100Ω resistors are connected first in star and then in delta across $415V$, 3ϕ supply. Calculate the line & phase current in each and also power taken from source.

Solution: -





जब Resistor star γ में जुड़ा है

$$\text{phase volt } V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415}{\sqrt{3}} = 239.6V$$

$$\text{phase current } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{100} = 2.396A$$

$$\text{Line current } I_L = I_{ph} = 2.396A$$

$$\begin{aligned} \text{Power drawn, } P &= 3 I_{ph}^2 R_{ph} \\ &= 3 \times (2.396)^2 \times 100 \\ &= 1722W \end{aligned}$$

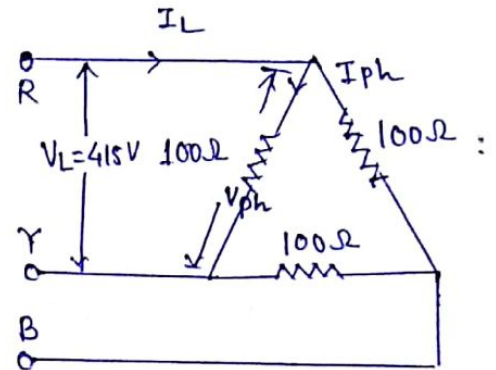
② जब Resistor Δ में जुड़ा है

$$V_{ph} = V_L = 415V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{415}{100} = 4.15A$$

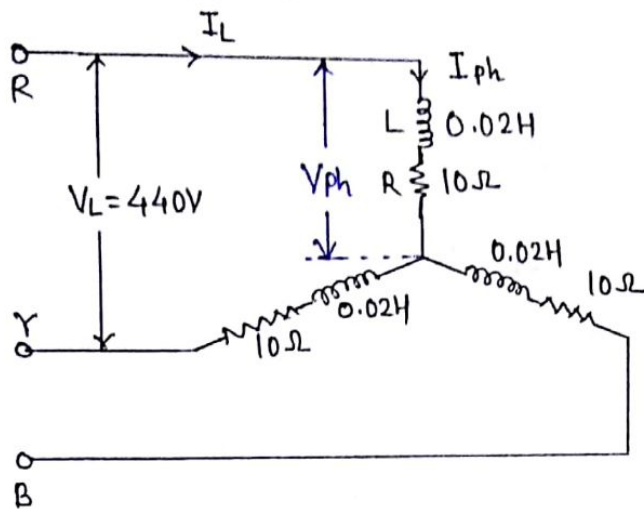
$$\begin{aligned} I_L &= \sqrt{3} \cdot I_{ph} \\ &= \sqrt{3} \times 4.15 = 7.188A \end{aligned}$$

$$\begin{aligned} \text{Power drawn } P &= 3 I_{ph}^2 \cdot R_{ph} \\ &= 3 \times (4.15)^2 \times 100 \\ &= 5166W \end{aligned}$$



Que: Three coils each having resistance of 10Ω and inductance of $0.02H$ are connected in star across $440V, 50Hz, 3\phi$ supply. Calculate line current and total power consumed.

Solution:-



$$\text{Inductive Reactance } X_L = 2\pi fL$$

$$= 2\pi \times 50 \times 0.02$$

$$= 6.283\Omega$$

$$\text{Impedance per phase } Z_{ph} = \sqrt{(10)^2 + (6.283)^2} \quad \therefore z = \sqrt{R^2 + X_L^2}$$

$$= 11.81\Omega$$

$$\text{Phase voltage} = V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254.03V$$

$$\text{Phase current } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.03}{11.81} = 21.51A$$

$$\text{Line current } I_L = I_{ph} = 21.51A$$

$$\text{Power factor, } \cos\phi = \frac{R_{ph}}{Z_{ph}} = \frac{10}{11.81} = 0.8467 \text{ Lag.}$$

$$\text{Total power} = \sqrt{3} V_L I_L \cos\phi$$

$$= \sqrt{3} \times 440 \times 21.51 \times 0.8467$$

$$= 13880W$$

$$= 13.88kW$$



Que: Three equal impedance each having a resistance of 8Ω and inductive reactance of 6Ω are connected in (a) star (b) delta across 3ϕ 440V system. Find:-
 (a) phase current (b) line current (c) Total power consumed

Solution: (a) star connection

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254V$$

$$Z_{ph} = \sqrt{R^2 + X_L^2} = \sqrt{(8)^2 + (6)^2} \\ = 10\Omega$$

$$(1) I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254}{10} = 25.4A$$

$$(2) I_{ph} = I_L = 25.4A$$

$$(3) \text{Power factor } \cos\phi = \frac{R}{Z} = \frac{8}{10} = 0.8 \text{ Lagging}$$

$$(4) \text{Power} = \sqrt{3} V_L I_L \cos\phi \\ = \sqrt{3} \times 440 \times 25.4 \times 0.8 \\ = 15.488 \text{ kW}$$

(b) Delta connection:-

$$V_{ph} = V_L = 440V$$

$$Z_{ph} = 10\Omega$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{440}{10} = 44A$$

$$I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 44 = 76.21A$$

$$\cos\phi = R/Z = 0.8 \text{ Lagg.}$$

$$\text{Power} = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 440 \times 76.21 \times 0.8 = 46.464 \text{ kW}$$

Q. A balanced star-connected load of $(8+j6)\Omega$ per phase is connected to a 3ϕ , 230V supply. Find line current, power and power factor.

Solution:-

$$\bar{Z}_{ph} = 8 + j6$$

$$= \sqrt{(8)^2 + (6)^2} \angle \tan^{-1}(6/8)$$

$$= 10 \angle 36.87^\circ$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.79V$$

$$\textcircled{1} I_L = I_{ph} = \frac{V_{ph}}{\bar{Z}_{ph}} = \frac{132.79}{10 \angle 36.87} = 13.279 \angle -36.87^\circ A$$

$$\textcircled{2} \text{Power factor, } \cos \phi = \cos 36.87 \\ = 0.8 \text{ lag — Ans.}$$

$$\textcircled{3} \text{Power} = \sqrt{3} V_L I_L \cos \phi \\ = \sqrt{3} \times 230 \times 13.279 \times 0.8 \\ = 4232 \text{ W — Ans.}$$

★ Consider Star Connection:-

चित्र के अनुसार Two wattmeter method की star connection में जोड़ा गया है।

Instantaneous current through current coil of $W_1 = I_R$

W_1 के current coil में Instantaneous current = I_R

W_1 के Potential coil में instantaneous voltage = $E_{RN} - E_{BN}$

अतः W_1 में instantaneous power = $I_R (E_{RN} - E_{BN})$

W_2 के current coil में instantaneous current = I_Y

W_2 के Potential coil में instantaneous voltage = $E_{YN} - E_{BN}$

अतः W_2 में Instantaneous power = $I_Y (E_{YN} - E_{BN})$

$$\begin{aligned} \therefore W_1 + W_2 &= I_R (E_{RN} - E_{BN}) + I_Y (E_{YN} - E_{BN}) \\ &= I_R \cdot E_{RN} - I_R \cdot E_{BN} + I_Y \cdot E_{YN} - I_Y \cdot E_{BN} \\ &= I_R \cdot E_{RN} + I_Y \cdot E_{YN} - E_{BN} (I_R + I_Y) \\ &\quad \therefore (I_R + I_Y + I_B = 0) \\ &= I_R \cdot E_{RN} + I_Y \cdot E_{YN} - E_{BN} (-I_B) \\ &= I_R \cdot E_{RN} + I_Y \cdot E_{YN} + I_B \cdot E_{BN} \\ &= \text{Total power absorbed by three load} \\ W_1 + W_2 &= P \end{aligned}$$

★ TWO WATTMETER METHOD (BALANCED LOAD)

Wattmeter Reading $W_1 = V_L I_L \cos(30 - \phi)$

Wattmeter Reading $W_2 = V_L I_L \cos(30 + \phi)$

दोनों Wattmeter reading को जोड़ने पर

$$\begin{aligned} W_1 + W_2 &= V_L I_L \cos(30 - \phi) + V_L I_L \cos(30 + \phi) \\ &= V_L I_L [\cos(30 - \phi) + \cos(30 + \phi)] \end{aligned}$$



$$\begin{aligned}
 W_1 + W_2 &= V_L I_L [\cos 30 \cos \phi + \sin 30 \sin \phi + \cos 30 \cos \phi - \sin 30 \sin \phi] \\
 &= V_L I_L [2 \cos 30 \cos \phi] \\
 &= V_L I_L \left(2 \times \frac{\sqrt{3}}{2} \cos \phi \right)
 \end{aligned}$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos \phi \quad \text{--- (1)}$$

$$W_1 + W_2 = P \text{ (Total power absorbed by } 3\phi \text{ balanced load)}$$

एथा

$$\begin{aligned}
 |W_1 - W_2| &= V_L I_L (\cos(30 - \phi) - \cos(30 + \phi)) \\
 &= V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)] \\
 &= V_L I_L [\cos 30 \cos \phi + \sin 30 \sin \phi - (\cos 30 \cos \phi - \sin 30 \sin \phi)] \\
 &= V_L I_L [2 \sin 30 \sin \phi] \\
 &= V_L I_L \left[2 \times \frac{1}{2} \sin \phi \right]
 \end{aligned}$$

$$W_1 - W_2 = V_L I_L \sin \phi \quad \text{--- (2)}$$

समी. (2) को समी. (1) से भाग देने पर

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{V_L I_L \sin \phi}{\sqrt{3} V_L I_L \cos \phi}$$

$$\sqrt{3} \left(\frac{W_1 - W_2}{W_1 + W_2} \right) = \tan \phi$$

$$\tan \phi = \sqrt{3} \left(\frac{W_1 - W_2}{W_1 + W_2} \right)$$

Que: A balanced 3 ϕ star connected load draws power from a 440V supply. The two wattmeter connected indicate $W_1 = 4.2$ kW and $W_2 = 0.8$ kW. Calculate power, P.f & current drawn from circuit.

Solution: $W_1 = 4.2$ kW

$$W_2 = 0.8 \text{ kW}$$

$$\begin{aligned} \therefore \text{Total power } P &= W_1 + W_2 \\ &= 4.2 + 0.8 \\ &= 5.0 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Now } \tan \phi &= \sqrt{3} \left(\frac{W_1 - W_2}{W_1 + W_2} \right) \\ &= \sqrt{3} \left(\frac{4.2 - 0.8}{4.2 + 0.8} \right) \end{aligned}$$

$$\tan \phi = 1.17$$

$$\begin{aligned} \phi &= \tan^{-1}(1.17) \\ &= 49.47^\circ \end{aligned}$$

② Power factor $\cos \phi = \cos(49.47) = 0.6472$

③ Now $P = \sqrt{3} V_L I_L \cos \phi$

$$5 \times 1000 = \sqrt{3} \times 440 \times I_L \times 0.6472$$

$$I_L = 10.137 \text{ A}$$

Que: Two wattmeters are used to measure the power in 3-phase balanced system. What is power factor when both the meter read equal (2) both the meter read equal but one is negative (3) One reads twice the other.



Solution: Case I: When both meter read equal, i.e. $W_1 = W_2$

$$\begin{aligned}\tan \phi &= \sqrt{3} \left(\frac{W_1 - W_2}{W_1 + W_2} \right) \\ &= \sqrt{3} \left(\frac{W_1 - W_1}{W_1 + W_1} \right)\end{aligned}$$

$$\tan \phi = 0$$

$$\phi = \tan^{-1}(0) = 0^\circ$$

$$\text{Power factor} = \cos \phi = \cos 0^\circ = 1 \text{ — Ans.}$$

Case II: अब दो readings समान हैं किंतु एक -ve है: अर्थात्

$$W_1 = -W_2$$

$$\tan \phi = \sqrt{3} \left(\frac{-W_2 - W_2}{-W_2 + W_2} \right)$$

$$\tan \phi = \infty$$

$$\phi = \tan^{-1} \infty = 90^\circ$$

$$\text{Power factor} = \cos \phi = \cos 90^\circ = 0 \text{ — Ans.}$$

Case III: अब एक ही मीटर का reading दूसरे से double है

$$W_1 = 2W_2$$

$$\tan \phi = \sqrt{3} \left(\frac{2W_2 - W_2}{2W_2 + W_2} \right)$$

$$= \sqrt{3} \times \frac{1}{3}$$

$$\tan \phi = \frac{1}{\sqrt{3}} \quad \phi = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = 30^\circ$$

$$\text{Power factor} = \cos \phi = \cos 30^\circ = 0.866 \text{ — Ans.}$$